Activity description

This activity uses examples in the context of shape and space to:

- find the upper and lower bounds of measurements, given the level of accuracy used
- consider how possible errors accumulate in formulae that involve measurements.

Suitability

Level 2 (Intermediate/Higher).
This activity could also be useful in supporting the study of science and engineering at Level 3.

Time

2–3 hours

Resources and Equipment

Combined student information sheet and worksheet

Optional: slideshow
Calculators

Key mathematical language

Uncertainty, error, mistake and accuracy need to be clarified.
Decimal places, significant figures, maximum, minimum, upper and lower bounds

Notes on the activity

Before using this activity, students will need to know how numbers are rounded to given levels of accuracy. They will also need to know how to find the perimeter of shapes, the area and circumference of circles, and the area of rectangles and triangles. They must also be able to substitute values into other given formulae and use Pythagoras' Theorem.

The accompanying slideshow can be used to introduce the activity and demonstrate the examples on the student sheets. You may decide to split the activity into two parts, by leaving Section B which deals with the accumulation of errors in calculations until a second session.

During the activity

Students could work individually, but the work involving calculations can be done more quickly if they work in pairs, with one student working out the lower bound and the other student working out the upper bound.

Points for discussion

Discuss how taking a measurement can lead to inaccuracy, and the variety of ways used to indicate how accurate a measurement is. Emphasise that when a measurement is expressed to a given unit, the maximum error is half this unit. Include each of the following:

Accuracy of measure	**Maximum error**
Nearest 100	50
Nearest 10	5
Nearest whole number	0.5
To 1 decimal place (nearest 0.1)	0.05
To 2 decimal places (i.e. nearest 0.01)	0.005

Unless students have studied errors before, they are likely to assume that answers to calculations involving measurements are much more accurate than they are.

The examples in Section B show how errors accumulate. Discuss the results with students, and introduce the usual convention of rounding the final answer to the same degree of accuracy as that of the least accurate measurement.

The examples show that even this is optimistic and it is possible, though unlikely, that the actual result may be different from the final answer given.

The 'Reflect on your work' section at the end of the student sheets (and on the last slide) can be used to aid class discussion at the end of the session.

Extensions

Able students could move on to consider situations involving subtraction or division (such as speed) or percentage errors.

Answers

Errors (page 4)

	Largest possible error	Upper bound	Lower bound
Height of a tree	0.5 m	50.5 m	49.5m
Mid-day temperature	0.5°C	28.5°C	27.5°C
Weight of a letter	0.5 g	32.5 g	31.5 g
Time to complete task	0.5 min	40.5 min	39.5 min
Length of caterpillar	0.05 cm	3.45 cm	3.35 cm
Patient's temperature	0.05°C	38.65°C	38.55°C
Weight of parcel	0.05 kg	2.95 kg	2.85 kg
Time to reach 60 mph	0.05 s	6.25 s	6.15 s
Length of shelf	0.005 m	2.755 m	2.745 m
Weight of fish	0.005 kg	1.645 kg	1.635 kg
Sprint time	0.005 s	10.275 s	10.265 s
Height of a hill	5 m	485 m	475 m
Width of drive	5 cm	565 cm	555 cm
Weight of cake	5 g	1205 g	1195 g
Weight of cake	50 g	1250 g	1150 g
Length of a runway	50 m	1950 m	1850 m
Length of a runway	5 m	1905 m	1895 m
Weight of an aircraft	5000 kg	175 000 kg	165 000 kg
Weight of an aircraft	500 kg	170 500 kg	169 500 kg

Combining errors (page 8–9)

1a i 260 m **ii** UB = 262 m, LB = 258 m
b i 4176 m^2 **ii** UB = 4241.25 m^2, LB = 4111.25 m^2

2 UB (circumference) = 17.31 cm, LB (circumference) = 17.25 cm
UB (area) = 23.84 cm^2, LB (area) = 23.67 cm^2

3a i 8.89 m^2 **ii** UB = 8.93 m^2, LB =8.86m^2
b i 2.05 m **ii** UB = 2.05 m, LB = 2.04 m

4a 89 797 mm^3 **b i** 91 735 mm^3 **ii** 87 887 mm^3

5a i 13.5 m^3 **ii** UB = 13.6 m^3, LB = 13.4 m^3
b i 31.4 m^2 **ii** UB = 31.6 m^2, LB = 31.2 m^2

6a Maximum area = 76.9 cm^2, minimum area = 74.5 cm^2
b Maximum perimeter = 35.1 cm, minimum perimeter = 34.5 cm